

§ 2.1 Superconformal Multiplets and Supersymmetric Deformations

$$\{Q_i, Q_j\} \sim 0, \quad i, j = 1, \dots, N_Q$$

Superconformal multiplet: $Q^\ell \mathcal{V}$,

where $0 \leq \ell \leq l_{\max}, \quad l_{\max} \leq N_Q$

Long multiplets

no null-states

\mathcal{V} is irreducible under Lorentz- and R-sym.

at level ℓ : $(\Lambda^\ell R_Q) \otimes \mathcal{V}$ (x)

where R_Q is the Lorentz- and R-sym rep of Q

→ unique top component: $l_{\max} = \dim R_Q = N_Q$

→ Lorentz- and R-sym. singlet

dimension: $\Delta_{\mathcal{V}} + \frac{1}{2} N_Q$

- Racah-Speiser algorithm :

- select highest-weight state $V_{h.w.} \in \mathcal{V}$
with respect to L-and R-sym.

- at each level ℓ , consider all sequences of ℓ supercharges acting on $V_{h.w.}$

$$Q_i, Q_{i_2}, \dots, Q_{i_\ell} V_{h.w.}$$

add L- and R-sym. weights of Q_i to

those of $\mathcal{V}_{n,w}$.

→ set of RS trial weights $w_{RS}^{(\ell)}$ at level ℓ

→ one-to-one correspondence between irreducible
reps. of G and RS trial weights $w_{RS}^{(\ell)}$

- when V is too small, the bijection can fail

Example:

consider long multiplets in 3d $N=1$ SCFTs

→ R-sym. is trivial, L-sym. is $su(2)$

supercharges $Q_\pm (\pm = \pm)$ transforms L-doublet

notation: $R_Q = [1]$

If L-rep. of V is $[n] \rightarrow n$ -index symmetric

spinor $\mathcal{V}_{(\alpha_1, \dots, \alpha_n)}$ with $\alpha_{i=1, \dots, n} = \pm$

$$\ell=0 : \quad \mathcal{V}_{n,w} = \mathcal{V}_{+++}, \quad w_{RS}^{(0)} = \{[n]\}$$

$$\ell=1 : \quad Q_+ \mathcal{V}_{n,w}, Q_- \mathcal{V}_{n,w}, \quad w_{RS}^{(1)} = \{[n+1], [n-1]\}$$

$$\ell=2 : \quad Q_+ Q_- \mathcal{V}_{n,w}, \quad w_{RS}^{(2)} = \{[n]\}$$

For $n \geq 1$, we have a match between $w_{RS}^{(\ell)}$ and
irreps of conformal primaries

For $n=0$ only $[1]$ rep. appears at level 1

$[-1]$ is removed by RS algorithm

RS trial states do not coincide with true highest weight states of corr. rep. :

true highest weight at $l=1$ of $[n-1]$ is

$$Q_- V_{++\dots+} - Q_+ V_{-++\dots+} \neq Q_- V_{++\dots+}$$

(RS algorithm bypasses full Clebsch-Gordan problem)

$$Q_- V_{+++ \dots +} = \frac{1}{2} \left[\text{highest weight}([n-1]) + \text{h.w.}([n+1]) \right]$$

$\rightarrow Q_-$ does not take us from $[n]$ to $[n-1]$

transition $l \rightarrow l+1$:

suppose \mathcal{O} is conformal primary at level l

$Q\mathcal{O} \rightarrow \mathcal{O}'$ at level $l+1$

$$\mathcal{O}' \subset R_Q \otimes \mathcal{O}$$

However: \mathcal{O}' might not be in the image of Q !

- 1) due to Fermi-statistics or if multiplet has null-states
- 2) \mathcal{O}' occurs at level $l+1$, but transition does not occur!

example:

consider long multiplet in 2d $N=4$ SCFT

$\rightarrow Q_\infty^{i,i}$ transform in trifundamental $\{1\}_{1/2}^{(4,1)}$

of $\text{su}(2)_L \times \text{su}(2)_R$ and $\text{su}(2)$ L-sym.

take $\nu \in [0]^{(0,0)}$ to be a singlet

true highest weights:

$$l=2: S = (Q_+^{++} Q_-^{--} + Q_-^{-} Q_+^{++} - Q_+^{+-} Q_-^{-+} - Q_+^{-+} Q_-^{+-}) \nu$$

$$l=3: G = Q_+^{++} S \in [1]^{(1,1)}$$

$$l=4: G' = Q_+^{++} Q_+^{+-} Q_-^{-+} Q_-^{++} \nu \in [2]^{(2,2)}$$

Note that $Q: G \rightarrow G'$ does not occur because

$$Q_+^{++} G = (Q_+^{++})^2 S = 0 \text{ by Fermi statistics}$$

$$\text{although } [2]^{(2,2)} \in \mathcal{R}_Q \otimes G = [1]^{(1,1)} \otimes [1]^{(1,1)}$$

short multiplets:

possess null states \rightarrow must be removed

from representation

We have the following possibilities for top components:

1) "Manifest top components":

conformal primary G with $QG = \text{descendant}$

i.e. no conformal primary at level $l+1$

in tensor product $\mathcal{R}_Q \otimes G$

examples: - all primaries at level l_{\max}

- universal mass deformation in 3d
discussed in last lecture

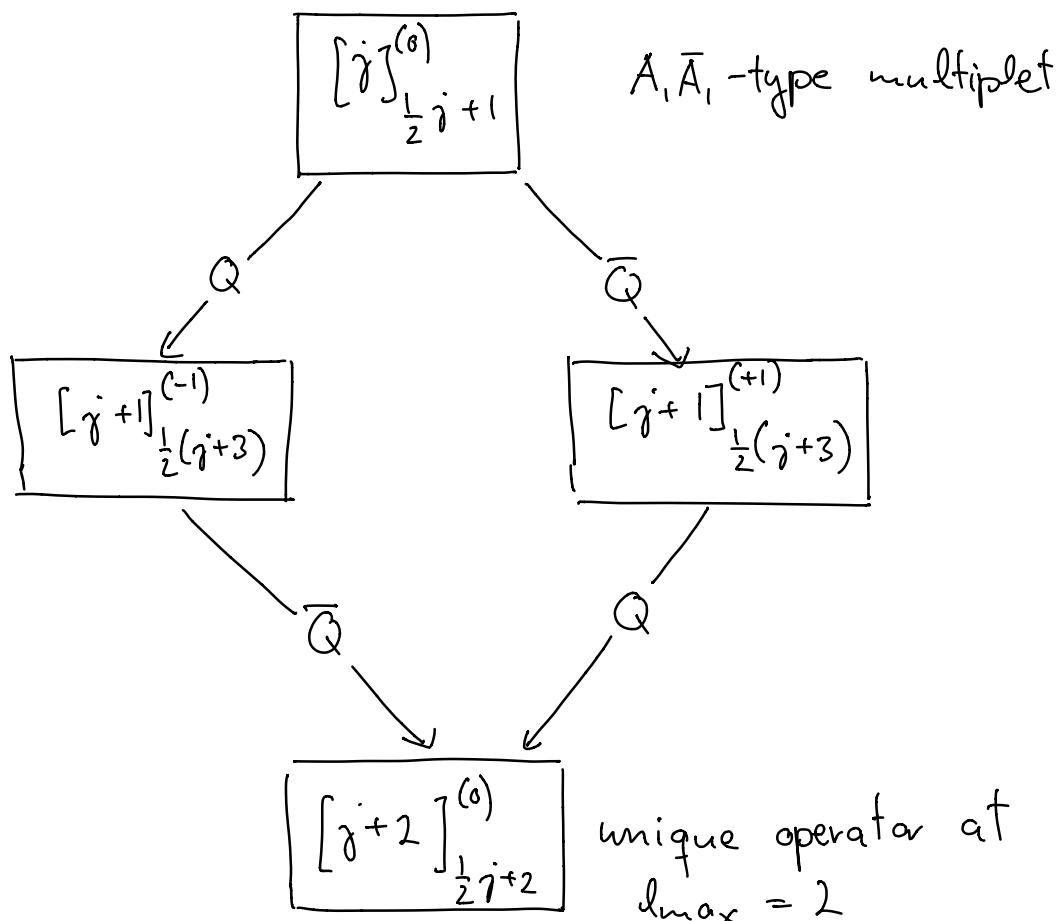
2) "Accidental top components":

there are conformal primaries at level $l+1$
 but they not in the image of $Q|_{\text{level } \ell}$
 We will focus on class 1)

Consider a generic short multiplet in 3d $w=2$

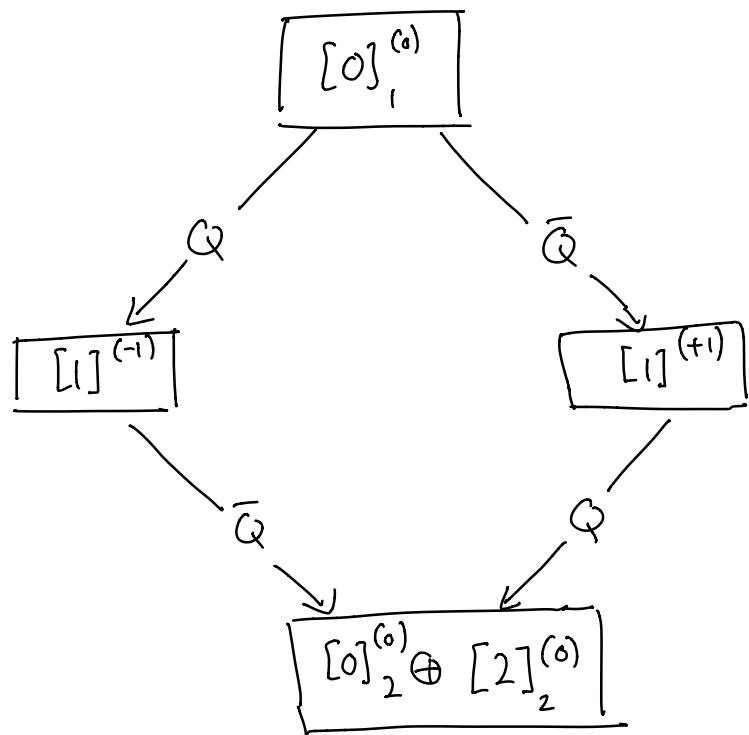
Q_α, \bar{Q}_α carry $U(1)_R$ charges -1 and +1

$$\rightarrow [1]_{1/2}^{(-1)} \oplus [1]_{1/2}^{(+1)}$$



case $j=2 \rightarrow$ superconformal stress tensor

example with two top components $A_2 \bar{A}_2 [0]_1^{(0)}$:



$[2]_2^{(0)}$: conserved flavor current

$[0]_2^{(0)}$: Lorentz-invariant flavor mass deformation